

Exercise 1.5.5

(\leq, F) is totally ordered field

$A \subseteq F$ and ℓ (lower band) of A exists.

Since F is complete $\sup A$ exists

Suppose we have $B \subseteq F$,
and all elements of B
are smaller than or equal to all
elements of A .

Then $\sup B = \inf A$, hence
 $\inf A$ exists

More formally

$$B = \{b \in F \mid b \leq a \text{ for all } a \in A\}$$

B is non-empty since A is lower banded
 B is banded above (all elements of A

$\sup B \leq a$ for all $a \in A \Rightarrow$ it a lower bound of A

If c is a lower band of A , then $c \leq \sup B$
so $\sup B$ is the greatest lower bound